Bulgarian Journal of Agricultural Science, 17 (No 2) 2011, 218-231
Agricultural Academy

INDIRECT ESTIMATION OF FARM'S RISK AVERSION: MATHEMATICAL PROGRAMMING APPROACH

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Abstract

ZGAJNAR, J. and S. KAVCIC, 2011. Indirect estimation of farm's risk aversion: mathematical programming approach. *Bulg. J. Agric. Sci.,* 17: 218-231

In the paper we present an approach how farmer's risk aversion could be estimated indirectly. This is particularly beneficial if one analyses hypothetical farms with absence of decision makers, as for example in the case of representative farms that are usually used for systematic studying. Applied approach is based on mathematical programming methods. The main idea is to use current farm practice as a baseline and to calculate missing data with partial optimization process. Non-interactive procedure based on expected value-variance framework and quadratic programming paradigm minimising variance has been applied to locate current farms' plans in expected value - variance space and to estimate their risk aversion. To demonstrate applicability of the approach, three representative dairy farms were analysed. Obtained results indicate high relative risk aversion in all three cases. More intensive dairy farm in flat area is less risk averse as smaller, still intensive farm with similar production conditions or farm with less intensive farming typical for hilly area. The study illustrates also discrepancy between optimal solutions considering or neglecting farmers risk aversion.

Key words: risk aversion, quadratic programming, livestock farms, mathematical programming
Abbreviations: CAP – Common Agricultural Policy; CARA – Constant Absolute Risk Aversion; CE – Certainty Equivalent; DARA – Decreasing Absolute Risk Aversion; E – Expected value; E(U(X)) – expected utility of final wealth; EUMV – Expected Utility Mean Variance; FADN – Farm Accountancy Data Network; IARA – Increasing Absolute Risk Aversion; k – capitalization factor; LP – Linear Program; MAUT – Multi Attribute Utility Theory; MCDM – Multi Criteria Decision Making; r_A – absolute risk aversion; r_R – relative risk aversion; SD – Standard Deviation; V – Variance; W – Wealth; Y – short period income

Introduction

Agricultural production planning is on different stages of production process connected with risk and uncertainty. Both are present in all business sectors and they should be considered when decisions are drown, no matter if these are decisions taken on everyday management basis, or projects (ones in a life time) that have significant impact on overall business performance. Common to all such kind 'business' decisions is that they are made for the future that could be more or less

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better predicted (based on expected values and different scenarios – states of nature). In other words, production decisions have to be taken without knowing the future state of nature, but on the basis of expectations concerning frequency of occurrence of different states of nature. Beside common sources of risk like in other sectors (price risk, market risk, institutional risk etc.), nature, in all broaden context, is another important source that occupies agriculture with additional risk that significantly influences the outcome of production. For some sources of risk we presume to know probabilities (frequency of states of natures) and levels of outcomes, while for the other this is not the case. This is one of the crucial points in risk modelling that defines which group of the methods should be applied. Pecher and Hahn (1999) are stating, if the probability of occurrence for an event is quantifiable, the situation is defined as risky and if not as uncertainty.

Since the purpose of this analysis is to find the optimal production plan for representative dairy farms, it is necessary to consider decision maker's propensity to accept risk. It measures the willingness to accept or not accept possible reduction in expected result. This is also one of the main points where agricultural producers disagree on it. In the literature many studies have been presented analysing characteristics as age, level of education, size of the farm, material status, sex etc., that significantly influence degree of risk aversion (Just, 2003; Meyer and Meyer, 2006)). Gardebroek (2006) compared risk attitudes of organic and non-organic farms. Lybbert and Just (2007) have analysed the relation between risk aversion and wealth on the basis of different probabilities.

From descriptive point of view, risk attitude parameters could be divided into absolute and relative. Absolute risk aversion (r_A) could be mathematically calculated as second and the first derivative of utility function (Meyer and Meyer, 2006; Hardaker et al. 2007). Consequentially, all utility functions representing a particular decision maker's risk preferences result in exactly the same absolute risk aversion (Meyer and Meyer, 2006). Absolute risk aversion can be interpreted as the percentage change in marginal utility. The coefficient r_A takes either positive or negative values. For farmers that are assumed to be risk averse it is negative. There are different situations how the coefficient behaves. Notwithstanding, it is denominated as coefficient, it is a function of wealth (Hardaker et al., 2007). Hardaker et al. (2007) are stressing that it is generally accepted that it behaves inversely as wealth. However, it could either decrease as allocably value increase (decreasing absolute risk aversion; DARA) or increases (IARA) as wealth increases. The first relation has become well accepted, while the second one has been rejected, since observed and predicted situations didn't match (Meyer and Meyer, 2006; Escalante and Rejesus, 2008; Saha et al., 1994). If the coefficient does not change parallel with monetary change expressed in wealth or other argument, in one or the other way described above, we have to deal with constant absolute risk aversion (CARA).

If the functional form is CARA, it means that an economic agent does not consider the level of the argument (wealth, income) of the utility function by taking risky decision (Gomez-Limon et al. 2003). Hardaker et al. (2007) are stressing that this simplification of constant coefficient holds under assumption that the change caused by risk is relatively small comparing to (total) initial or permanent wealth. If the model is applied to decision-makers with significant differences in initial wealth, the CARA behavioural construct should be replaced by DARA (Escalante and Rejesus, 2008). It is generally accepted that the absolute risk aversion coefficient will decrease with increase in wealth since people can easier afford to take risk if they are richer (Hardaker et al., 2007).

r_A is a non-dimensional measure of risk aversion. Its value is dependent on the currency in which the monetary units are expressed and could therefore not be compared between farms originating from different countries with different currency

(Gomez-Limon et al., 2003; Hardaker et al., 2007). Similar problem occurs if wealth is measured in nominal terms in different points at time or if rate of return is measured in decimal instead in percentage form (Meyer and Meyer, 2006). Common to all three pointed examples is 'in-consistency' across units in which the outcome variable is measured. This drawback could easily be mitigated with another function developed by Pratt and Arrow more than fifty years ago. It is a function of relative risk aversion (Meyer and Meyer, 2006).

The relative risk aversion (r_R) measures the elasticity (slope) of the marginal utility in terms of the percentage change in the monetary variable (Gomez-Limon et al. 2003). The risk aversion coefficient is exogenously specified (by the farmer or decision maker) and as long as it is not known, some range of relative risk aversion (scenario analysis) might be used for modelling (Ogurtsov, 2008). Anderson and Dillon (1992) proposed a classification of the relative risk aversion coefficient. It ranges between 0.5 for slightly risk averse economic agents up to 4 for extremely averse ones. Normally risk averse agent has coefficient of 1, rather risk averse 2 and value 3 indicates very risk averse individual. In the literature there are also examples where relative risk aversion might be even higher. Meyer and Meyer (2006) are reporting that magnitude of relative risk aversion range up to values approaching one hundred. For example Gomez-Limon et al. (2003) are calculating risk aversion in the context of *multi attribute utility theory* (MAUT), where monetary and non-monetary attributes are considered and the values of r_R are ranging higher as 25. On the basis of non-parametric approach Lien (2002) has calculated values that range up to 10.80. Saha et al. (1994) have found that magnitude of r_R at the mean level of wealth is between 3.8 and 5.4 depending on the sample used in the estimation. Meyer and Meyer (2006) are explaining that a substantial part of this variation is due to the differences in the definition of outcome variables applied in different analysis. They are making a point on different defining of wealth, income etc. Even if the difference seems to be minor, it could lead to significantly different r_R measures that are not directly comparable. Hardaker et al. (2007) are stressing that if the argument of the utility function is something else than wealth or income, for example gross margin per unit, it is difficult to assess an appropriate degree of risk aversion that would be consistent.

In decision analysis, r_R is usually taken as a basis for r_A estimation (Ogurtsov, 2008). Under assumption that r_R is more or less constant for local variation in wealth, r_A could be estimated using the formula (1) (Hardaker, 2000). This relation is especially important if in risk analysis r_A is needed, like in linear mean-variance approach. However, either multiplication or division can be used to convert one measure into another.

$$r_A(W)= r_R(W)/W \qquad (1)$$

When r_A has to be estimated on the basis of relation between r_R and wealth (described by upper relation) it is important to consider under what assumptions we are going to estimate r_A. Beside change in wealth discussed before, it is also important to distinguish between transitory income and permanent income as the argument of utility function (Hardaker et al., 2007). Permanent income should be considered when the uncertainty is about long-run level of income, while transitory income should be taken as the argument when the income is uncertain in the near future (next year or couple of years). The latest is also the case in our model since results obtained are expressed in one year time horizon (annual income per farm and gross margin per activities). The difference is in certain value of capitalization factor – k ($k>1$; $k=W/Y$) that increases short period income (Y) comparing to long time horizon income (for more details see Hardaker et al. 2007).

The proper selection – estimation of risk aversion is crucial to find the optimal production plan for the farmer or to locate farms decision margins in expected value – variance (E,V) space. A variety of methods have been developed to measure the risk attitudes of agricultural producers (Antle, 1987). From the literature three different generally known aspects could be husked (Gomez-Limon et al. 2003). One is direct estimation of the utility function (direct interaction with the decision-maker), the second approach is 'experimental method' and the third is observation approach by tuning the models to fit actual behaviour - data. For the last group parametric (econometric) as well as non-parametric (mathematical programming) approaches are applied to observe relationship between actual behaviour of agents and the one predicted from models. Parametric approach is presented by Saha et al. (1997), who applied Expo-Power utility function that has ability of exhibiting decreasing, constant or increasing absolute risk aversion and decreasing or increasing relative risk aversion. An example of econometric estimation of producers' risk attitude is also Antle (1987), who presents methodology for estimating the distribution of risk attitudes in a population of producers utilizing similar production technologies. On the other hand non-parametric approach is applied by Gomez-Limon et al. (2003), who present new approach to estimate risk aversion inside the MCDM paradigm. Lien (2002) is another example of non-parametric estimation of risk aversion values based on imitating actual farmers' behaviour.

The traditional approach to the problem of modelling production process under risk is the expected utility model (EU model). It focuses on the stochastic production function and estimation of probability distribution of yield and prices (Rasmussen, 2004). In this paper mean-variance approach proposed by Freund (1956) will be applied, derived from the portfolio problem as discussed by Markowitz. Even though E, V method has been criticised by many agricultural economists as a deficient method, Gomez-Limon et al. (2003) are citing some papers that justify its use. Crucial conditions that should be satisfied are that argument of utility function is normally distributed and that risk involved is small in comparison with the total wealth. Mean-variance models are consistent with expected utility maximization only under quadratic utility or normality (Just, 2003).

The paper proceeds as follows. The next section outlines the approach applied to estimate risk aversion parameter followed by short review of literature on E,V modelling. Then the approach for estimating degree of risk aversion is precisely described with description of the model and analysed dairy farms, detailed specification of natural endowments, production orientation and the current production situation. The subsequent section presents results with focus on estimated risk aversion coefficients and location of farms in E,V decision space. The paper concludes with short discussion and some ideas how to proceed in further modelling.

Material and Methods

In this study it is assumed that farmers are risk averse, maximize their utility. Consequentially concave form of utility function has to be applied, what in mathematical terms means that the first derivative of utility function is positive and the second derivative is negative (Lien et al., 2009). In principle any kind of utility function satisfying these conditions might be used (Lien et al., 2009).

For modelling farm production under risk, as the problem of maximizing expected utility, generalized mean-variance approximation to expected utility could be used (Escalante and Rejesus, 2008). To justify its use, two assumptions are necessary to be satisfied. First is that distribution of the outcome variable (argument of the utility function) is (approximately) normally distributed[1] and the second that farmer's utility function could be represented by a negative exponential utility functional form. Consistent with Freund's (1956) results, due to normal distribution, which means that distribution is completely specified by the

(1) This assumption could be drawn on the basis of Central Limit Theorem due to the fact that total outcome is the sum of several random variables – combination of different activities (Hardaker et al., 2007).

mean and variance, instead of negative exponential function, expected utility is expressed as a simple function of expected value (E) and variance (V) (Hardaker et al., 2007). Traditional EUMV (expected utility mean variance) decision model used in this analysis is presented in Equation 2.

$$\text{Max } E(U(x)) = E(x) - 0.5r_A V(x) \qquad (2)$$

where E(U(x)) is the expected utility of final wealth (outcome variable), E(x) is the expected (or mean) final wealth, V(x) is the variance of final wealth. We have converted the expected utility of net income to an estimate of the certainty equivalent (CE). The estimates of CEs are readily interpreted because they are expressed in monetary units (Hardaker et al., 2008).

The mean-variance utility function (2) implicitly assumes constant absolute risk aversion (Nelson and Escalante, 2004; Hardaker et al 2007; Escalante and Rejesus 2008). Escalante and Rejesus (2008) are stating that such model produces reasonable solutions and behavioural predictions for situations when changes in outcome variable (income) is relatively small comparing to total value (wealth or income). This is also the main fact that justifies its use in our analysis, since expected changes caused by risk are relatively small in comparison to total farm wealth. This fact would be different if the model would include also high risky investment activities (e.g. investment in biogas plant station for electricity production or bigger investments in infrastructure – stable, machinery), which would significantly change the production structure and consequentially also total farm wealth. However, this presumption defends also our decision that utility and risk aversion coefficients are not measured in wealth, but as annual (transitory) income. Namely, bad or good results on annual basis has no significant effect on wealth and hence on income levels in subsequent years (Hardaker et al., 2008).

To find the optimal solution on the efficient curve, indifference curve has to be plotted in E, V space. It slope defines coefficient, known as risk aversion. For this purpose a non-interactive modelling approach has been applied, based on mathematical model representing farmers' decision behaviour. The main idea of applied approach is to observe actual farmers behaviour without use of questioners or other direct instruments. Applied methodology has been presented and developed by Lien (2002). In our paper Lien's approach has been slightly adopted in the phase of current farm situation estimation (partial-optimization) and by calculating two points on the efficient curve (simple regression analysis).

Approximation of decision maker's absolute risk aversion coefficient proposed by Lien (2002) is to estimate two points on E, V efficient frontier. Efficient curve could be derived from points calculated by minimising variance (*V*), while expected income (EI_f) is parameterized (3),

$$\begin{aligned} &\min V = \sum_{j=1}^{n}\sum_{k=1}^{n}\sigma_{jk}x_j x_k; \quad s.t. \\ &\sum_{j=1}^{n} c_j x_j - FC = \lambda EI_f \\ &\sum_{j=1}^{n} x_j a_{ij} \le b_i \\ &x_j \ge 0;\ x_k \ge 0 \end{aligned} \qquad (3)$$

or by maximizing expected income (*EI*), while variance (V_f) is varied over its feasible range (4). Both approaches should yield identical solutions.

$$\begin{aligned} &Max\ EI = \sum_{j=1}^{n} c_j x_j - FC \quad s.t. \\ &\sum_{j=1}^{n}\sum_{k=1}^{n}\sigma_{jk}x_j x_k = \lambda V_f \\ &\sum_{j=1}^{n} x_j a_{ij} \le b_i \\ &x_j \ge 0;\ x_k \ge 0 \end{aligned} \qquad (4)$$

Notations in mathematical models 3 and 4 have the following meaning:

V – minimal total variance

V_f – parameterized total variance ($\lambda \geq 0$) when the model is used to formulate E,V efficient curve or observed variance when current production practice ($\lambda = 0$) at the farm is analysed

EI – expected income

EI_f – parameterized expected income ($\lambda \geq 0$) when the model is used to formulate E,V efficient curve or observed expected income at current farm production practice ($\lambda = 0$)

c_j – expected gross margin per unit of activity

x_j – activities in the model

a_{ij} – technical coefficients

b_i – constraints of the model

FC – fixed costs

σ_{jk}- variances and covariances between activities

λ - parameterization factor

Further, observed current farm practice (situation) is plotted in E, V space. This is simply done by calculating expected income (EI_F) and expected variance (V_F) for the current farm situation. In most cases it is expected that this (E, V) pairs lie under the curve, while Lien (2002) reports that it might also happen that for some farms this is not the case. She is pointing on some possible reasons why this might happen.

Next step is to calculate more efficient solutions (lying on the E, V curve) in the sense of variance and expected income for analysed farm. This is done by minimizing variance ($V_{E,V}$) for the observed expected income (EI_F) and the second by maximizing expected income ($E_{E,V}$) at the level of variance observed at the farm (V_F). For this step Lien (2002) quadratic models (3) and (4) has been applied (in both mathematical models parameterization factor (λ) has not been considered). First approach is an example of minimizing total variance (3) and the second one, maximizing expected value where variance enters as constraint (4). As these two points are determined in E, V space, r_A as the gradient of the line between them could be calculated (5).

$$r_A = \frac{2(E_{E,V} - E_F)}{V_F - V_{E,V}} \tag{5}$$

What has been added to the Lien's (2002) approach is partial constrained optimization in the first step in which current situation at the farm is estimated. The main problem is that not all the data for this step are available. The reason is either in aggregated data or they are simply not available. Namely, on the basis of FADN data different farms have been formulated representing typical average dairy farms in Slovenia (Rednak et al., 2009). The fact is that in this way constructed farms have relatively exact structure of 'final tradable activities', but lack of information on other on-farm activities. In the case of livestock production, as the core subject of this paper, this means that we do not have the exact information on the extent of fodder production activities (it has been estimated in the process of defining farms). To overcome this drawback we decided to include partial optimisation. Our basic idea is that missing data (information) could be calculated on the basis of deterministic linear programming model, maximizing expected income (*EI*) as partial optimization process (6) to (9).

$$\max EI = \sum_{j=1}^{n} c_j x_j + \sum_{f=1}^{r} c_f x_f - FC \quad s.t. \tag{6}$$

$$\sum_{j=1}^{n} a_{ij} x_j + a_{if} x_f \leq b_i \tag{7}$$

for all i = 1 to m

$$x_f = x_{farm} \tag{8}$$

$$x_j \geq 0 \tag{9}$$

Partial term is connected with available data (number of breeding animals, selling/purchasing activities, maintenance of arable land etc.) that are available and known and therefore also fixed (x_{farm}) in the optimization process with additional constraints (8) added into the model. Optimization is therefore performed just for those activities (x_j)

on grassland (how much is gathered as hay, silage, pasture etc.) where only intensity of production is known and for fodder purchasing activities.

This additional step means also that ‘observed solution’ would be technical more efficient and consequentially the discrepancies would be smaller (due to optimised part), but we also ensure that situations are comparable. As a result, all farm production plans would lie under efficient curve, which was not the case by Lien (2002).

Further we have adjusted Lien’s (2002) approach in the step of calculating two points on efficient curve necessary to calculate r_A (5). First we have applied quadratic model in E,V framework, minimising variance (3) to formulate efficient frontier (approach is precisely described by Zgajnar and Kavcic (2010); the difference is only that income is used instead of gross margin as argument of function). Further its mathematical equation was estimated with a tool for fitting curves from MS Excel. It is an example of simple regression analysis that gives beside curve’s equation also R^2.

In the next step estimated equation for the efficient curve has been used to calculate both (more efficient) pair wise points $V_{E,V}$ and $E_{E,V}$ necessary in equation (5). First point ($V_{E,V}$) presents pair with farm’s expected income (current situation) and minimal variance calculated from estimated equation. The second point ($E_{E,V}$) consists of variance observed at the current farm practice and maximal expected income that could be achieved at such level of variance. Above described approach has been applied due to the fact that Lien’s approach in our case did not always give solutions on the E, V curve. Our opinion is that this has happened due to solver ‘strength’ problems. Namely, in our analysis MS Excel common nonlinear solver has been applied that seems to be less efficient as some other solvers applied in other studies (Premium Solver or GAMS –CONOPT etc.).

Data and farm description

All the data required for ‘feeding’ the model have been obtained from additional simulation model that calculates technical coefficients as well as economic arguments for pre-selected activities. Activities could be therefore easily adjusted to the observed farm situation (technology and quantity of production). Set of historical data (ten year time series) prepared by Slovene Agriculture Institute (KIS, 2009) have been used as a source of risk for decision variables in the model.

The main principle applied in our analysis is to find the optimal production plan for a planning horizon of one year. This means that farmer could decide what to produce at the beginning of each year and he/she is deciding on the basis of his/her expectations of returns (expressed as income per farm and as expected gross margin per activity) of production at the end of the year. No investment activities are presumed. Expectations are based on expected (average) gross margins and their variances and co-variances, calculated on the basis of historical data. The set of historical data has been updated for possible changes in technology utilizing ‘de-trending’ process detailed described by Hardaker et al. (2007). Further price deflator index has been applied to convert nominal prices to a real (2008) prices.

In ten year period different occasions (extreme draught in 2003; beneficial climate conditions in 2004 and 2005; CAP reform etc.) have occurred that have manifested in variations - in our model expressed as risk. Due to these ‘outer’ triggers we have assigned different probabilities to each state of nature (ten years horizon) to reflect the chance that similar conditions will prevail in the planning period. Regarding expert opinion, the most possible (0.16) state of nature are conditions (prices, costs and yields) in 2008, while conditions in 1999 are less probable (0.06).

In the proper risk analysis each individual farm would have to have own characteristics in the sense of variance, representing production circumstances, technologies, natural conditions and efficiency of the farm (Lien, 2002). In other words this would mean that for each farm different variance-covariance matrix should be formed. Unfortunately this is not the case in risk modelling. Beside lack of data (time series) concerning current activities applied at the farm, there is another problem concerning activities that have not yet been applied and consequentially no 'historical' data are available (Lien, 2002). Time series data are therefore the same for similar farms (homogenous group) with similar technologies, size, production conditions and natural endowments. In our model this means that variance-covariance matrix is drown from the same source of (historical) data, but is adjusted for each analysed (typical) farm with different pre-selected activities and the level of production (output). Therefore also results obtained should be taken as explanation of typical behaviour.

Activities and constraints of analysed dairy farms

Three representative dairy farms were chosen to reflect production endowments in Slovenia. The main activities and constraints, included in the modelling, are the following:

(i) Livestock activities include two technologies with dairy cows (intensive farming with Holstein Frisian breed and less intensive practice with Simmental breed), bulls fattening (that starts at 90 kg, with 1.1 kg average daily gain and stops at 750 kg live weight) and heifers breeding activity that presumes potential female calf purchase and selling of pregnant heifers.

(ii) Activities on arable and grassland include selling crop activities (maize, wheat and barley) on arable land as well as fodder production (wheat, barley, maize, maize silage, maize-grain silage, grass silage produced on grassland and on arable land as greening cover (ensilaged into silo and bale), hay (dried on the meadow or using drying system on cold air) and pasture). Activities on grassland could be divided into two groups. In the first group activities with the same yield and number of cutting (2, 3 or 4) through the whole year could be merged. And in the second group 15 activities with different gathering technologies on the same area through the year could be merged. Also in this group selection between 2, 3 or 4 cutting technologies could be chosen.

(iii) Purchasing activities for concentrated feed (17) with different levels of energy, proteins and quality. The model could also include purchasing of maize, wheat or barley if necessary. In this group also activities for hiring labour (dispersed over four periods) and renting land (arable and grassland) might be classified.

(iv) Transfer and endogenous activities concerning crop rotation on arable land, forage conservation technologies on grassland and subsidy activities. The latest are those CAP measures that are not coupled with production activities (arable and grassland regional (area) payments).

The main endowments - constraints of analysed farms are presented in Table 1. The first farm is an example of high intensive specialised dairy farm, selling calves and heifers and without beef production. Significant part of the ration presents fodder from arable land. Similarly to farm 1, also farm 2 operates in flat area, but with less intensive technology. Third farm is typical dairy farm in Slovenian hilly area with less intensive production and important share of fodder produced on grassland. Beside milk production beef is important source of income at this type of farm.

Labour demand is divided into four three-month periods. We presumed that family labour available is spread within four periods 20%, 30%, 30% and 20% respectively, while additional labour could be hired at times of working peaks. In all three analysed cases it is presumed that labour could be hired up to 30 % of own labour capacity and not additionally constrained within periods. The same holds for arable and grassland, where farmer could rent only the same category (in the sense of quality and type) of land as he/she owns. This assumption was made because if more land is required transport cost would significantly increase as well as different machinery would be necessary and the calculations should be changed. The same logic has been taken by determining expected yield for new activities explained later in the text.

Additional constraint is added to ensure that not significant share of the land available is left idle (for grassland it is presumed maximum 20% and for arable land maximum 10%). The model includes also constraints connected with crop rotation on arable land (maximal share of maize, wheat, barley, clover and minimal share of clovers). The same constraints are presumed for all three farms. In Table 1 current production structure on arable land is presented. In optimization process the model can include also other activities from the list of activities. The main guidance was that only those activities could be selected that has similar attributes as activities already practised on the farm. Such new activities could enter as fodder production activities or as cereals for selling. Intensity of production was adjusted according to production achieved by maize production.

Due to differences in natural conditions by farms (slop of the area, dry/wet area, average climate conditions for the region etc.) the model includes additional constraints on grass utilisation (maximal shares of hay, silage, pasture etc.). These conditions are different for analysed farms. The main difference is in grass silage (bale or silo), hay production (on meadow or on drying system) and pasturing that is possible only by the third farm. From Table 1 it is also apparent that due to different natural conditions, different technologies (number of cutting) are presumed.

All three farms are representative livestock

Table 1
Main characteristics of analysed farms

Resources		Farm 1	Farm 2	Farm 3
Labour available	(hours)	8 100	3 240	2 700
Tillage area				
Arable land	(ha)	27	8	4
Grassland	(ha)	22	9	10
Activities on arable land				
Maize grain	(ha)	9	2	0
Maize silage	(ha)	8	2	2
Green cover	(ha)	10	4	2
*No. of mowing on grassland**		4 (3)	3 (4)	2 (3)
Livestock activities				
Dairy cows				
Breed		HF	HF	SIM
Milk yield per cow		8 000	7 800	6 900
Current number	(heads)	57	21	16
Heifers breeding				
Current number	(heads)	20	7	3
Bulls fattening				
Current number	(heads)	0	0	11
Depreciation	(€)	16 900	6 813	5 972

* The first number relates to majority of grassland area and the second to the rest.

farms oriented in dairy production. First two breed high productive Holstein Frisian cows, while the third one is an example of less intensive dairy breed production. In all three cases it is presumed that suitable proportion of female calves are bred to pregnant heifers and the rest female and male calves are sold, except the third farm where production plan includes also bulls fattening.

Since animal requirements are expressed at the level of nutrients (crude proteins, metabolic energy, net energy for lactation, dry matter, minimum and maximum crude fibre) the model includes also the so called 'balance constraints' ensuring that animal requirements are fulfilled and that the surplus of nutrients is in reasonable quantities.

Subsidies are important income of EU farms. In the model subsidies are taken as they were in the year 2008. Our hypothetical assumption was that no major changes are going to happen in the years to come. This means that farms are entitled to regional payments (for grassland and arable land) and are in the model included as additional activities, while production coupled bulls special premium is included (add up) at the level of calculations made per activity (historical data set). This means that subsidies directly influence the expected income.

Fixed costs presented in Table 1 are presumed to be certain for observed short term planning horizon. No investments activities are included and also infrastructure needs are presumed to have enough capacities for foreseen extent of production. Accounted fixed costs (FC in equation 4) include just machinery and farm building depreciation cost.

Results

The stated purpose of this study was to estimate degree of risk aversion for different dairy farm types in Slovenia maximizing their utility. Farms analysed were taken from another study (Rednak et al., 2009) in which typical farms were constructed from FADN data. We decided to focus just on

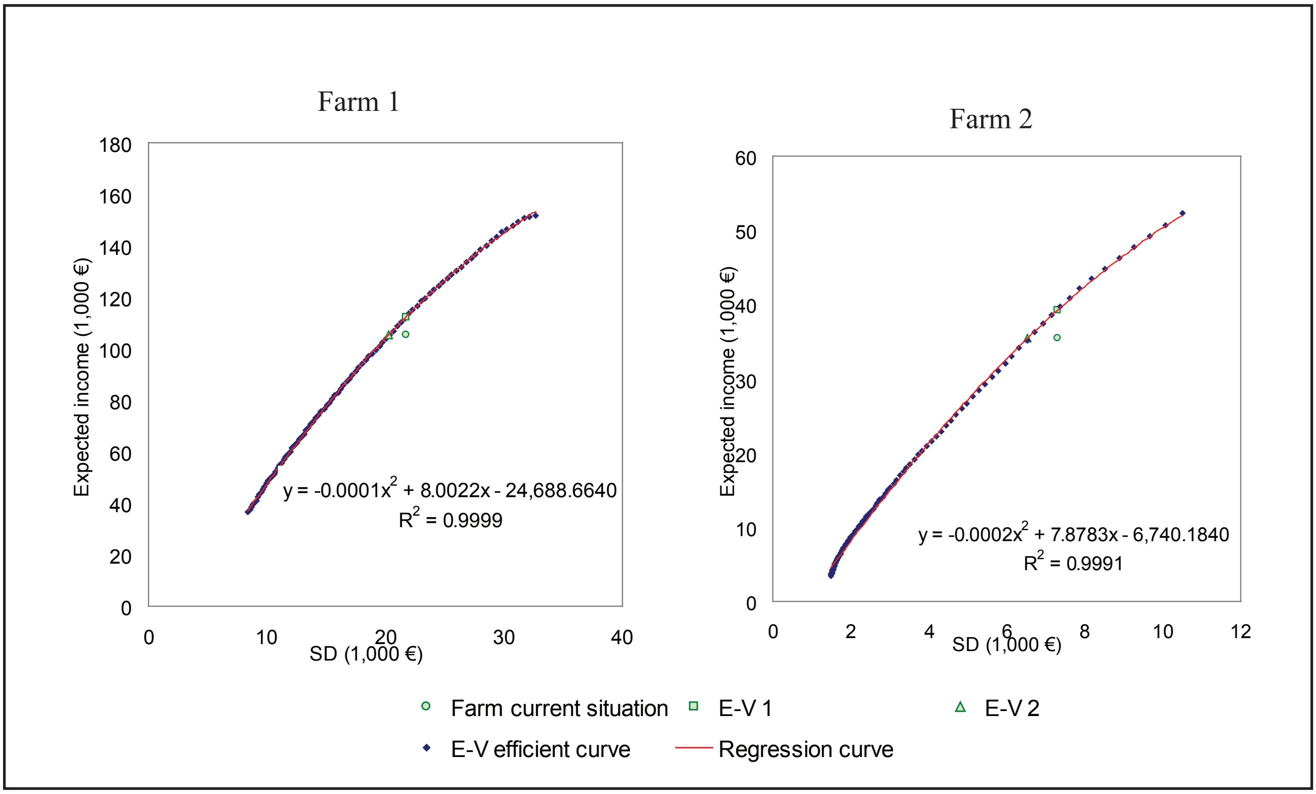

Fig. 1. Farm current situation and efficient points E-V 1 and E-V 2 plotted in the E,V space

three typical dairy farms that are representative for Slovene dairy production.

The main purpose of the study was to develop a tool based on presented methodology to analyse farmers' attitude toward risk. In the first part of this section most important intermediate results necessary to estimate absolute risk aversion are presented (Table 2). It is followed by obtained risk aversion behaviour indicators. In the last part of this chapter, two efficient curves, with all three points defining observed farmers' behaviour are illustrated (Figure 1).

From Table 2 it is apparent that farm 1 achieves the highest income which is due to natural endowments, presented in Table 1, not surprising. However with just more efficient production plan, farm could improve income for almost 7,000 € or almost 6.5% of current realisation. On the other hand the same income could be achieved for almost the same relative decrease (6.7%) in total variance as a measure of risk. Lien (2002) compares efficiency and inefficient portfolios in the E, V space with technical efficiency from efficiency and productivity literature. It reflects the ability of a farm to obtain the maximum output from a given level of inputs. From this point of view farm 2 and 3 are technically less efficient, since discrepancy between actual and expected income is larger (more than 10% by the third farm).

To show the fact that consideration of risk is very important in analysing optimal production structure, we present also solutions of common deterministic LP. It presumes that manager is risk neutral and focuses just on maximal income achieved. Significant better result could be achieved in all three cases. At this point it also has to be noted that in all three cases it was presumed that additional land and labour could be rented (maximal 30% of own capacities), however all the rest endowments and constraints are the same. The biggest difference occurs by the third farm, where deterministic solution achieves 47.06%

Table 2

Current economic performance on farms, more efficient solutions and coefficients indicating farmers' attitude to risk

		Farm 1	Farm 2	Farm 3
Current situation (F)				
Income	(€)	105 542	40 295	35 524
SD	(€)	21 759	8 694	7 306
Maximal income (E e,v)				
Income	(€)	112 424	44 06	39 247
SD	(€)	21 759	8 694	7 306
Minimal variance (V e,v)				
Income	(€)	105 542	40 295	35 524
SD	(€)	20 299	7 905	6 542
Maximal income (LP - solution)				
Income	(€)	151 715	57 88	52 242
SD	(€)	32 677	12 499	10 497
Estimated wealth	(€)	110 000	45 000	40 000
Risk parameters				
rA		0.0002242	0.000575	0.000704
rR(W)		24.662	25.869	28.169

improvement in expected income. Of course this improvement is related also with significant risk increase, measured as standard deviation (SD). From Table 2 it is apparent that SD increase ranges between 43.67 and 50.18% respectively. Therefore it is very important to consider risk in searching for optimal farm plan; otherwise obtained solution is misleading for farmer as well as for policy makers if such model is used also for their purposes. Namely, by neglecting risk the model would yield a solution that demands' taking on risk the farmer is not prepared to accept. In other words the production plan does not reflect farmer's behaviour. Therefore it is very important to consider problem as stochastic and to know where on the efficient curve solution is acceptable for the farmer.

At the beginning of the paper different approaches to find acceptable and achievable range on the efficient curve for particular farm were presented. On the basis of location of current farm plan into the E,V space, 'acceptable' range for the farmer has been defined and his absolute risk aversion has been estimated (Table 2).

For individual case farms absolute risk aversion with respect to transitory income vary from 0.0002242 to 0.000744. It is apparent that the lowest (0.0002242) absolute risk aversion is achieved by the first farm, while more risk averse are the second and the third farm. The relevant relative risk aversion, defined as marginal utility of wealth, was derived from absolute risk aversion. As regards from estimated wealth and expected income, relative risk aversion ranges up to 24.7 for the first farm, 25.9 for the second and 28.2 for the third farm respectively (Table 2). On the basis of these results it could be concluded that the largest farm is less risk sensitive, while smaller farm in 'hilly' area is more risk averse – risk elastic. As stressed by Meyer and Meyer (2006) one has to bear in mind that by interpreting and comparing obtained results with related studies, any measure of risk aversion for farmer is specific to the particular outcome variable over which the measure is estimated. This might be the reason why coefficients are much higher for our case study, since our 'income' does not include all items as by the definition of income (no taxes, no insurance costs etc.). Meyer and Meyer (2006) are stressing that even if differences seem to be minor, they can led to significantly different estimates of the risk aversion measure. Even though one of the main purposes of this paper was to estimate farmers risk aversion, it is also very important to get information where in the E, V space current farm plan and consequentially also expected behaviour could be located.

For the first and the third farm whole efficient curves in E, V space are presented (Figure 1). For each farm efficient curve was derived by minimising variance and parameterization of expected income. As described in the previous chapter, MS Excel tool has been applied to adjust regression curve to the solutions plotted in E, V space. From R^2 it is apparent that estimated equations sufficiently describe optimised farm plans. If this coefficient wouldn't be that high, than just relevant (narrower) part of the E, V efficient curve would be taken into analysis. However this was not a problem in analysing all three farms as could be also seen for two of them from Figure 1.

On the basis of estimated equation both points on the E, V curve have been calculated. From Figure 1 it is apparent that for both farms segments on the curve, defined by calculated points, belong to the upper half of the E, V curve. The same holds also for the second farm which results are not presented in this paper.

Discussion

In comparison with other studies obtained relative risk aversion are relatively high. They do not correspond to Anderson and Dillon's (1992) proposed classification of the relative risk aversion (0 to 4). According to literature review (Meyer and Meyer, 2006; Hardaker et al. 2007) this has happened due to different (not proper) income definition was used. Therefore it is very important to know, how outcome variable is defined and what is considered and what is not. Meyer and Meyer (2006) are stressing that if one knows relations between different outcome variables or their utility functions, he could also gather on difference between risk aversion measures. However, the aim of this paper was not only to estimate relative risk aversion of farms and to compare them, but also to estimate absolute risk aversion and to develop a simple approach to be applied in further work on the rest of representative farm types modelled by Rednak et al. (2009).

Beside different production endowments that farms are confronted with, the difference between them occurs also in efficiency in selling strategies, searching market paths, purchasing strategies, collaboration with cooperatives, manner of payment etc.. The main common effect of this few listed facts are different achieved price for selling goods as well as for purchasing inputs. Consequentially it would be very beneficial in further research to incorporate some additional information in aggregated historical data. One of possibilities would be to apply approach presented by Hardaker et al. (2007) to reconstruct historical data into 'synthetic' data series. The main idea is that expected gross margins per activities and belonging standard deviation are expertly defined for each particular farm, while the correlation and other stochastic dependencies embodied in the original historical data remain the same.

Conclusions

The purpose of this paper was to investigate farms risk aversion. The main idea was to estimate risk aversion coefficient without direct interaction with farmer. Namely, the aim was to analyse some farm types that are representative for larger group of farms and tend to be 'model' farms for systematic studying, type like: what is going-on on particular farm, what kind of effect might have some policy measures, technology change etc. For this purpose complex tool has been developed, based on mathematical programming paradigm. Approach proposed by Lien (2002) has been slightly changed and adapted, especially due to the fact that MS Excel as a basic platform has been utilised.

It proved that the non-linear solver has sometimes problems in searching for optimal solution. Namely, obtained optimal solutions did in some cases not lie on E, V efficient frontier. Therefore whole efficient curve has been calculated by parameterization of expected income and minimizing variance. Further its equation has been estimated and two points necessary calculated.

The main advantage of approach presented in this paper is its simplicity. It has proofed useful also with less powerful non-linear solver.

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Received June, 2, 2010; accepted for printing January, 12, 2011.